Exercise 76

Given $f(x) = x^3 - 5$ and $g(x) = \sqrt[3]{x+5}$:

(a) Find $f(g(x))$ and $g(f(x))$.

(b) What does the answer tell us about the relationship between $f(x)$ and $g(x)$?

Solution

Calculate $f(g(x))$ by plugging the formula for $g(x)$ where x is in the formula for $f(x)$.

$$f(g(x)) = \left(\sqrt[3]{x+5}\right)^3 - 5$$
$$= (x+5) - 5$$
$$= x$$

Calculate $g(f(x))$ by plugging the formula for $f(x)$ where x is in the formula for $g(x)$.

$$g(f(x)) = \sqrt[3]{(x^3 - 5) + 5}$$
$$= \sqrt[3]{x^3}$$
$$= x$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, $f(x)$ and $g(x)$ are inverse functions.